$1+2+3+\cdots+n$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

"open form" (though I don't think anyone says that)

"closed form" (like 'formula')

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$$\lim_{n\to\infty} \frac{1+2+3+4+5+\cdots+(n-1)+n}{n^2}$$

$$= \lim_{n\to\infty} \frac{\left(\frac{n(n+1)}{2}\right)}{n^2} = \lim_{n\to\infty} \frac{n(n+1)}{2n\cdot n} =$$

$$= \lim_{n\to\infty} \frac{n+1}{2n} =$$

$$= \lim_{n\to\infty} \frac{1+\frac{1}{n}}{2} = \frac{1}{2}$$

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$$\lim_{n\to\infty} \sum_{k=1}^{n-1} \frac{2k^2}{n^3}$$

$$= \lim_{n\to\infty} \frac{2}{n^3}\left(\left(\sum_{k=1}^{n} k^2\right) - n^2\right)$$

$$\left(1^2+2^2+3^2+\cdots+(n-1)^2+\cancel{n^2}\right)\cancel{-n^2}$$

$$= \sum_{k=1}^{n-1} k^2$$

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$$\lim_{n\to\infty} \sum_{k=1}^{n-1} \frac{2k^2}{n^3} = \lim_{n\to\infty} \frac{2}{n^3} \boxed{\sum_{k=1}^{n-1} k^2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n-1} k^2 = \frac{(n-1)(n-1+1)(2(n-1)+1)}{6}$$

$$= \frac{(n-1)(n)(2n-1)}{6}$$

$$= \frac{2n^3-3n^2+n}{6}$$

$$\frac{n(n+1)(2n+1)}{6} - n^2$$

$$= \frac{2n^3+3n^2+n-6n^2}{6}$$

$$= \frac{2n^3-3n^2+n}{6}$$

Dec 17-7:32 AM

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$$\lim_{n\to\infty} \sum_{k=1}^{n-1} \frac{2k^2}{n^3} = \lim_{n\to\infty} \frac{2}{n^3} \sum_{k=1}^{n-1} k^2$$

$$= \lim_{n\to\infty} \frac{2}{n^3}\left(\frac{2n^3-3n^2+n}{6}\right)$$

$$= \lim_{n\to\infty} \frac{1}{3}\left(2-\frac{3}{n}+\frac{1}{n^2}\right) = \frac{2}{3}$$

$$= \frac{(n-1)(n)(2n-1)}{6}$$

$$= \frac{2n^3-3n^2+n}{6}$$

$$\frac{n(n+1)(2n+1)}{6} - n^2$$

$$= \frac{2n^3+3n^2+n-6n^2}{6}$$

$$= \frac{2n^3-3n^2+n}{6}$$

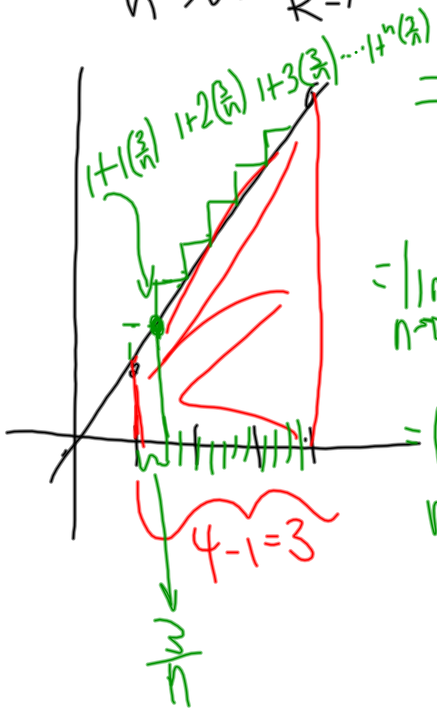
37) $A = \lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} f(x_k^*)\Delta x$ (6.4.3)

$y = \frac{1}{2}x$

$a = 1$

$b = 4$

$\frac{1}{2}$ —



$= \lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \left(\frac{1}{2}\left(1+k\left(\frac{3}{n}\right)\right)\right)\left(\frac{3}{n}\right)$

$= \lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \frac{1}{2}\left(1+\frac{3}{n}k\right)\left(\frac{3}{n}\right)$

$= \lim\limits_{n\to\infty} \frac{3}{2n} \sum \left(1+\frac{3}{n}k\right)$

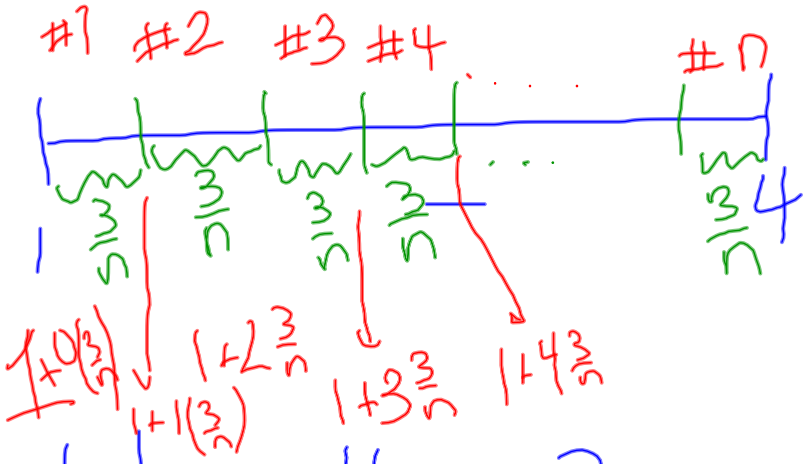
$= \lim\limits_{n\to\infty} \frac{3}{2n} \sum\limits_{k=1}^{n} \left(1+\frac{3}{n}k\right)$

$= \lim\limits_{n\to\infty} \frac{3}{2n} \left(\sum\limits_{k=1}^{n} 1 + \left(\frac{3}{n}\right)\sum\limits_{k=1}^{n} k\right)$

$= \lim\limits_{n\to\infty} \left[\frac{3}{2n}\left(n + \frac{3}{n}\left(\frac{n(n+1)}{2}\right)\right)\right]$

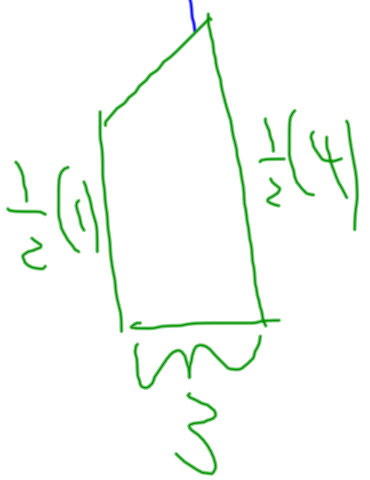
$= \lim\limits_{n\to\infty} \frac{3}{2} + \frac{9}{2n^2}\left(\frac{n(n+1)}{2}\right)$

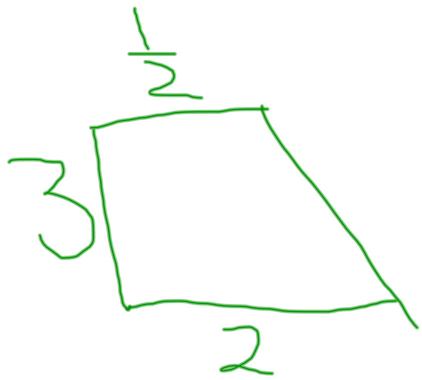
$= \lim\limits_{n\to\infty} \frac{3}{2} + \frac{9}{4}\left(\frac{n+1}{n}\right) = \frac{3}{2} + \frac{9}{4} = \frac{15}{4}$



total length $= 4-1=3$

divide into n pieces $=$ each length $\frac{3}{n}$





$$A_{\text{trap}} = \frac{1}{2}\left(\frac{1}{2}+2\right)(3)$$

$$\frac{1}{2}(2)(4) - \frac{1}{2}(1)\left(\frac{1}{2}\right) = \frac{1}{2}\left(\frac{5}{2}\right)(3) = \frac{15}{4}$$

$$4 - \frac{1}{4} = \frac{16}{4} - \frac{1}{4} = \frac{15}{4}$$

$$x_k^* = a + \boxed{(\quad)}\Delta x$$

$$y = \frac{1}{2}x^2$$